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2003 J. Phys. A: Math. Gen. 36 L7

(http://iopscience.iop.org/0305-4470/36/1/102)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **36** (2003) L7–L11 PII: S0305-4470(03)52837-2

LETTER TO THE EDITOR

Headway statistics of public transport in Mexican cities

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Received 28 August 2002
Published 10 December 2003
Online at stacks.iop.org/JPhysA/36/L7

Abstract
We present a cellular automaton simulating the behaviour of public bus transport in several Mexican cities. The headway statistics obtained from the model is compared to the measured time intervals between subsequent bus arrivals to a given bus stop and to a spacing distribution resulting from a random matrix theory.

PACS numbers: 05.65.+b, 45.70.Vn

Public transport in Mexico is organized differently to that in Europe. First, no leading companies are responsible for city transport. Thus, there are no timetables for city buses and sometimes not even well-defined bus stops. Moreover, the driver is usually the owner of the bus and so his aim is to maximize the income. Because every passenger boarding the bus has to pay, the driver tries to collect the largest possible number of passengers. When not regulated, the time interval during which two subsequent buses pass a given point will display a Poissonian distribution. This is a consequence of the absence of correlations between the motion of different buses. Such a situation is, however, not welcomed by the drivers because then the probability density that two buses will arrive at a bus stop within short time interval is large. In this case, the first bus collects all the waiting passengers and the second bus that arrives slightly later will find the stop practically empty. This simple reasoning makes it clear that the existence of certain correlations between buses, which will change the Poisson process, will be favourable. Indeed, in Mexico various strategies have been developed to create such correlations. Here we discuss the situation in three cities: Cuernavaca, Puebla and Mexico City.

In Puebla, there is no additive mechanism that helps to increase the bus correlation. Hence, the headway statistics of the buses in this city should be close to Poissonian. In Cuernavaca, in turn, information about the times when buses pass certain points are noted and then sold to the bus drivers (there is a real market with this information). The driver can, in such a way, change the velocity of the bus depending on the position of the bus in front of him and a bus–bus correlation appears.

To describe the correlations, we use a modified version of the celebrated Nagel–Schreckenberg cellular automaton (CA); see [1, 2]. Consider N equal cells on the line (for our purpose we define the length of one cell as 30 m) and n particles (buses) moving along it. Thus, $c = \frac{n}{N}$ is the bus density and $\bar{d} = c^{-1}$ is the mean distance between two neighbouring buses. Furthermore, we define the maximal velocity $v_{\max}$ of the bus and its probability p to slow down $p \in [0, 1]$. The Nagel–Schreckenberg model describes the dynamics of the system with the help of the following update rules. In time $t = 0$, the positions x_i of the particle $i = 1, \ldots, n$ are integer numbers randomly chosen from the set $1, \ldots, N$ satisfying the condition $x_{i-1} > x_i$ for every $i = 2, \ldots, n$. Furthermore, in time $t = 0$ the velocity v_i of the ith particle is set to zero for all i: $v_i(0) = 0$. The buses start to move with velocity v according to the update rules that have to be applied simultaneously to all particles.

- *Step one:* When the velocity of the bus is smaller than a maximal velocity $v_{\max}$, it increases its velocity by one
$$v_i(t+1) = \min\{v_i(t) + 1; v_{\max}\}; \tag{1}$$
- *Step two:* Particles with positive velocities are randomly slowed down
$$v_i(t+1) = v_i(t+1) - 1 \tag{2}$$
with probability p;
- *Step three:* Particles update their positions according to
$$x_i(t+1) = \min\{x_i(t) + v_i(t+1); x_{i-1}(t+1) - 1\} \tag{3}$$
i.e. the particles move according to the rule $x_i(t+1) = x_i(t) + v_i(t+1)$ with the restriction that they cannot occupy the same cell or overtake each other. In this case, the particle hops to the cell behind the occupied one.

To create a modification of the model and to adapt it to the situation in Mexico, we change the update rules on a subset M of possible bus positions where the information is passed to the driver. (The density of those points will be denoted by a: $a = M/N$.)

To change the model we add to steps (1), (2) and (3) an additive step (4) that takes into account the processing of the information.

At the points $j \in M$, the information about the time interval Δt to a preceding bus, which has passed that point, is available to the driver. Using this, the driver can modify the bus velocity; to speed up if $\Delta t > \bar{t}$ or to slows down for $\Delta t < \bar{t}$. (Recall that $\bar{t}$ is the mean time interval between subsequent buses.) Hence we change the model by adding

- *Step four:*
$$\begin{aligned} &v_j(t) = v_j(t) + 1 \\ \text{if } t_i(j) - t_{i-1}(j) < \bar{t} \qquad & \\ &v_j(t) = v_j(t) - 1 \end{aligned} \tag{4}$$
if $t_i(j) - t_{i-1}(j) > \bar{t}$ where $t_i(j)$ denotes the time when a bus i passed through the point j: $x_i(t_i(j)) = j$.

Using the modified model, we focus on the headway statistics, i.e. on the probability distribution of the time intervals Δt between two subsequent buses passing a given point and we compare it with the results obtained in the cities. For the simulation of the transport in Puebla, we choose $c = 1/40$, $v_{\max} = 2$, $p = 0.5$ and $a = 0$, taking into account the fact that

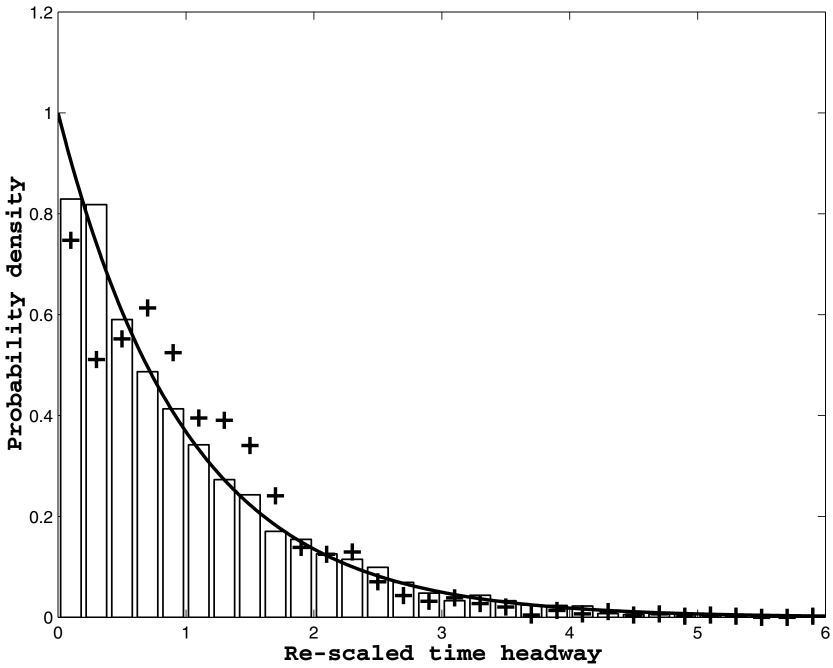

Figure 1. Time headway of the bus transport in Puebla. The curve represents the Poisson distribution (5). The crosses display the time headway distribution of buses in Puebla and the bars are taken from the CA model with $a = 0$.

there are no check points. It is not surprising that the headway statistics is in this case came close to the Poisson distribution (see figure 1):

$$P(t) = e^{-t} \tag{5}$$

when t is the spacing re-scaled to the mean value equal to one. Besides the headway statistics, we also compare a number variance $\Sigma^2(t)$ that is defined as

$$\Sigma^2(t) = \langle (n(t) - t)^2 \rangle \tag{6}$$

where $n(t)$ is the number of bus arrivals at a given point during the time period of the length t. Note that $\langle n(t) \rangle = t$ due to the fact that $\langle t \rangle = 1$. It can easily be checked that for a Poissonian process

$$\Sigma^2(t) = t. \tag{7}$$

The number variance obtained from the data and from the simulation fits quite well with this prediction (see figure 3). The data show, however, a small deviations from equation (7). This is a manifestation of a weak interaction between the buses, which probably originates from the fact that the buses interact through the number of passengers waiting on the stops. Namely, when the distance between the buses is large, more passengers are waiting at the stop and the bus is delayed longer at the stop.

To simulate the situation in Cuernavaca and in Mexico City, we use parameters $c = 1/40$, $v_{\max} = 2$, $p = 0.5$ and $a = 1/36$, which represent one bus per 1.2 km and one check-point per 1 km approximately. The modified CA leads, in this case, to significant changes in the time interval distribution (see figure 2). The distribution obtained from the

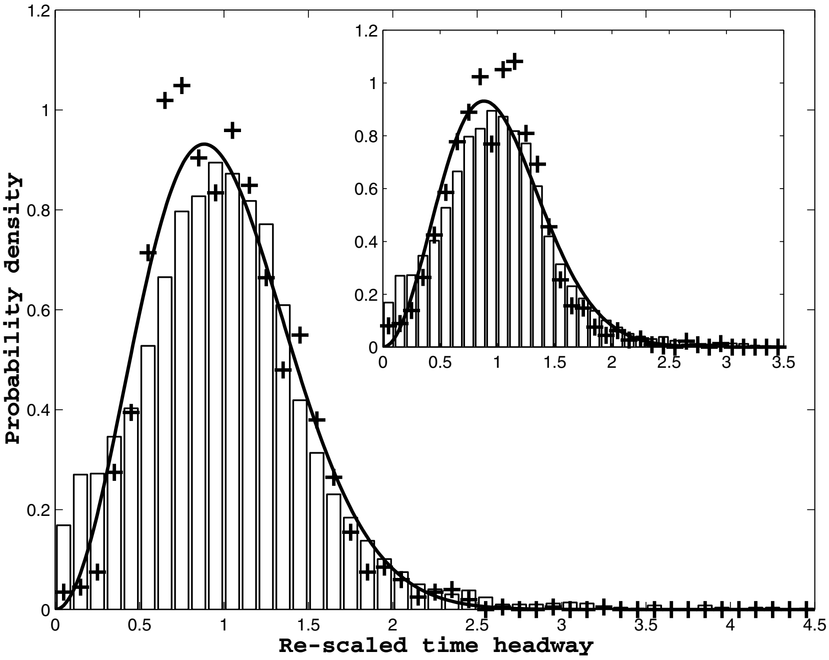

Figure 2. Time headway of bus transport in Cuernavaca and Mexico City. The curve represents the Wigner formula (8). The crosses display the time headway distribution of buses in Cuernavaca and the bars represent the results of the CA model with $a = 1/36$. The results obtained for buses in Mexico City are shown in the inset.

automaton fits well the observed time interval distribution. Moreover, both distributions conform well with the Wigner distribution (see [3]):

$$P(t) = \frac{32}{\pi^2} t^2 \,\mathrm{e}^{\frac{4}{\pi} t^2}. \tag{8}$$

It is well known that Wigner distribution describes the level spacing statistics of the Gaussian unitary ensemble of random matrices. The random matrix theory in turn describes the spacing distribution of certain one-dimensional interacting gas (Dyson's gas). So the link between the Wigner distribution and the headway statistics of the modified CA is not surprising.

Similar agreement is observed also for the number variance (see figure 3) where random matrix theory leads to

$$\Sigma^2(t) = \frac{1}{\pi^2}(\ln 2\pi t + \gamma + 1) \tag{9}$$

with $\gamma \approx 0.57\,721\,566$ (see [4]).

However, as evident in figure 3, the interaction between buses in Cuernavaca and in Mexico City is stronger than that resulting from the CA model, which leads to stronger correlations. Whereas in the automaton model correlations exist between the nearest neighbours only, in Cuernavaca and Mexico City we can recognize that interaction exists also between the first, second and third neighbours.

We conclude that the modified Nagel–Schreckenberg CA successfully describes microscopic properties of bus transport in some Mexican cities. The velocity of the buses is influenced by the information about their mutual positions so that the drivers can optimize their

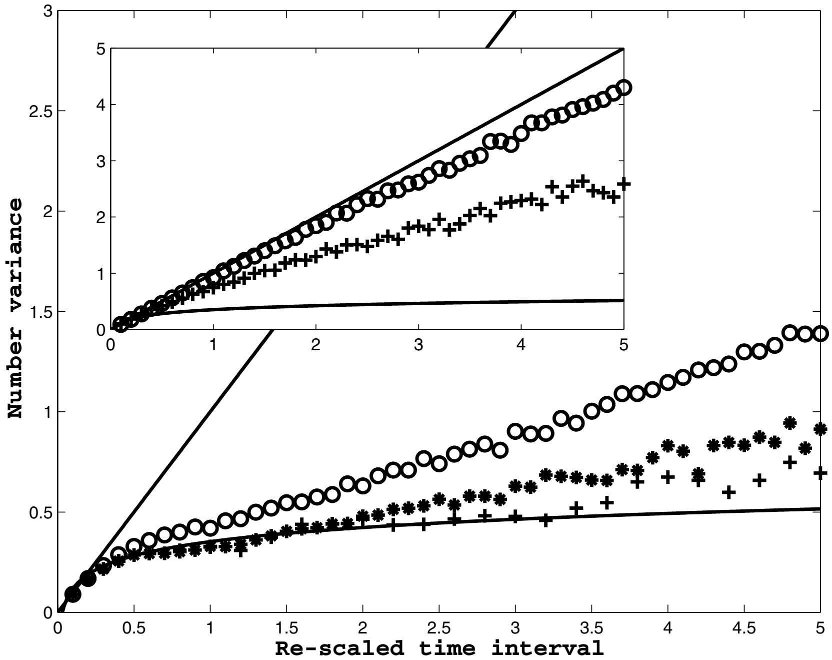

Figure 3. Number variance of the public transport in some Mexican cities. The curve and line represent the prediction of the random matrix theory (9) and (7), respectively. The crosses and stars display number variance obtained from public bus transport in Cuernavaca and Mexico City, respectively. The circles denote the results of the CA model for $a = 1/36$. The number variance obtained from the transport in Puebla is displayed in the inset (crosses). The circles show the results of the CA model obtained for $a = 0$.

rank in competing for the passengers to be transported. This finally increase the coordination of the bus motion and changes the time headway statistics.

Acknowledgments

This work has been supported by the grant A1048101 and GACR 202/02/0088 of the Czech grant agency. The data in the Mexican cities were collected with the friendly help of Dr Markus Mueller from the University in Cuernavaca and Dr Antonio Mendes-Bermudes from the University of Puebla. The authors are also very grateful to Patrik Kraus from the University of Hradec Králové for entering the collected data into the computer.

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